Question 1: (34 points) Circle the most correct answer in the following

1) The integral $\int_{\pi}^{\infty} \frac{1+\sin x}{x^2} dx$
   a) Converges by Direct Comparison Test with $\int_{\pi}^{\infty} \frac{2}{x^2} dx$
   b) Diverges by Direct Comparison Test with $\int_{\pi}^{\infty} \frac{2}{x^2} dx$
   c) Converges by Limit Comparison Test with $\int_{\pi}^{\infty} \frac{2}{x} dx$
   d) Diverges by Limit Comparison Test with $\int_{\pi}^{\infty} \frac{2}{x} dx$

2) The sequence $a_n = \sqrt[n]{4^n n}$
   a) Converges to 0
   b) Converges to 1
   c) Converges to 4
   d) Diverges

3) The series $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{5^n}$ is
   a) Convergent geometric series to $\frac{5}{6}$
   b) Divergent geometric series
   c) Convergent geometric series to $\frac{6}{5}$
   d) Divergent by n-th term test

4) One of the following statements is **false** about a series
   a) The sum of convergent and divergent series must be divergent
   b) The sum of two convergent series must be convergent
   c) The sum of two divergent series must be divergent
   d) The difference of two convergent series must be convergent

5) The series $\sum_{n=1}^{\infty} \frac{n^n}{3^{(2n+1)}}$
   a) Converges by the root test
   b) Diverges by the root test
   c) The root test fails
   d) Converges by ratio test

6) The infinite series $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$

a) Converges by ratio test
b) Diverges by ratio test
c) Converges by root test
d) Converges by n-th term test

7) The integral $\int_1^{\infty} \frac{1}{x^3-x} dx$

a) Converges by Direct Comparison Test with $\int_1^{\infty} \frac{1}{x^3} dx$
b) Converges by Direct Comparison Test with $\int_1^{\infty} \frac{1}{x} dx$
c) Converges by Limit Comparison Test with $\int_1^{\infty} \frac{1}{x^3} dx$
d) Diverges by Limit Comparison Test with $\int_1^{\infty} \frac{1}{x^3} dx$

8) The sequence $a_n = \frac{3^n \,.\, 6^n}{2^{-n} .\, n!}$

a) Converges to 0
b) Converges to 1
c) Converges to $\frac{1}{2}$
d) Diverges

9) The series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

a) Converges by sum partial test
b) Diverges Geometric series
c) Converges by n-th term test
d) Diverges by n-th term test

10) The series $\sum_{n=1}^{\infty}\left(\sqrt{n+4} - \sqrt{n+3}\right)$

a) Convergent telescoping series
b) Converges by n-th term test
c) Divergent telescoping series
d) Diverges by n-th term test

11) The sequence $a_n = \sqrt{n}\ \sin\left(\frac{1}{\sqrt{n}}\right)$

a) Diverges
b) Converges to 1
c) Converges to 0
d) Converges to $\frac{\pi}{2}$

12) One of the following statements is **true** about a sequence $\{a_n\}$

a) If $\{a_n\}$ is bounded then it converges
b) If $\{a_n\}$ converges then it is monotonic
c) If $\{a_n\}$ is converges then it is bounded
d) If $\{a_n\}$ is monotonic then it converges

13) The series $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^n$

a) Converges by n-th term test
b) Diverges by n-th term test
c) Converges by root test
d) Diverges by root test

14) If $a_1 = 2$, $a_{n+1} = \frac{2}{n} a_n$,then the series $\sum_{n=1}^{\infty} a_n$

a) Diverges by ratio test
b) Diverges by root test
c) Converges by ratio test
d) The ratio test fails

15) The series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

a) Converges by Limit Comparison Test with $\sum_{n=1}^{\infty}\frac{1}{n}$
b) Converges by n-th term test
c) Diverges by n-th term test
d) Diverges by Limit Comparison Test with $\sum_{n=1}^{\infty}\frac{1}{n}$

16) The infinite series $\sum_{n=1}^{\infty} n^{-2}$

a) Divergent p-series
b) Convergent by integral test
c) Convergent by the ratio test
d) Convergent by nth-term test

17) The series $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$

a) Converges by Direct Comparison Test with $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$

b) Diverges by Direct Comparison Test with $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$

c) Converges by Direct Comparison Test with $\sum_{n=1}^{\infty}(3)^n$

d) Diverges by Direct Comparison Test with $\sum_{n=1}^{\infty}(3)^n$

Determine if the following series converges or diverges

1) <u>Use The Integral Test</u>

8 points.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

the series has positive terms

let $f(x) = \frac{x}{x^2+1}$, $x \geq 1$

$f(x)$ is cont. , positive & decreasing

1 1 1

$$\int_1^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_1^b \frac{x}{x^2+1}\,dx = \lim_{b\to\infty} \frac{1}{2} \int_1^b \frac{2x}{x^2+1}\,dx$$

1

$$= \frac{1}{2} \lim_{b\to\infty} \ln|x^2+1| \Big|_1^b .$$

1

$$= \frac{1}{2} \lim_{b\to\infty} \left(\ln|b^2+1| - \ln(2) \right)$$

1

$$= \infty$$ Integral diverges

2 So, by the integral test, the series

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$ diverges.

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2)

8 Points

$$\sum_{n=1}^{\infty} \frac{n^2 (n+2)!}{n!\ 3^{2n}}$$

1 the series has positive terms

Apply Ratio test:

1 $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = $ 1 $\lim_{n\to\infty} \frac{(n+1)^2 (n+3)!}{(n+1)!\ 3^{2n+2}} \cdot \frac{n!\ 3^{2n}}{n^2 (n+2)!}$

1 $= \lim_{n\to\infty} \frac{(n+1)^2 (n+3)}{(n+1)\ 3^2\ n^2}$

1 $= \frac{1}{9} \lim_{n\to\infty} \frac{(n+1)(n+3)}{n^2}$

1 $= \frac{1}{9} < 1$

2 $\therefore$ by Ratio test the series $\sum_{n=1}^{\infty} \frac{n^2 (n+2)!}{n!\ 3^{2n}}$ converges.

8 points.

1) Given that the sequence $a_1 = 2,\ a_{n+1} = \frac{72}{(a_n)+1}$ converges. Find its limit

The seq. has positive terms, & it converges.

So, $\lim_{n\to\infty} a_n = L$ (1) & $\lim_{n\to\infty} a_{n+1} = L$ (1).

(1) $\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{72}{a_n + 1}$

(1) $L = \frac{72}{L+1}$ (1) $\Rightarrow L^2 + L - 72 = 0$

(1) $(L+9)(L-8) = 0$

(1) $L = -9$ ✗ $\boxed{L = 8}$ (1)

2) For what value of **r** does the series $\sum_{n=1}^{\infty} 2\, r^{n-1}$ converges to the number 5

5 points

it is a convergent Geometric series.

(1) with first term $a = 2$

& ratio $= r$.

(1) So, $\text{Sum} = \frac{a}{1-r}$

(1) $5 = \frac{2}{1-r}$

(1) $5 - 5r = 2 \Rightarrow \boxed{r = \frac{3}{5}}$

(1)